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A Contingency-Based Security-Constrained Optimal Power Flow Model For Revealing The Marginal Cost of a Blackout Risk-Equalizing Policy in the Colombian Electricity Market

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***Abstract*— This paper presents a new methodology to solve the dispatch problem in the Colombian electricity market. The proposed methodology is based on the use of coupled postcontingency Optimal Power Flows with additional linear constraints to include reliability criteria. The main contribution of this paper is the solution of the Colombian dispatch problem in one stage, which provides actual nodal prices and reveals the marginal price of a blackout−risk equalizing policy in the Colombian electricity market.**

***Index Terms*-- Power system scheduling, power system reliability.**

I. INTRODUCTION

THE problem of optimization in electric power systems appears from the moment in which two or more generators must supply several loads forcing the operator to decide how to distribute the load optimally among the different units. Historically, the first optimization efforts were made regarding generation control, what is known nowadays as classic economic dispatch [1]. The improvement of computer techniques, particularly the introduction of techniques for the efficient treatment of sparse matrixes and the development of new optimization algorithms, along with the greater capacity of computer processing have allowed the solution of problems that include network constraints and reliability criteria. This evolution has led to what nowadays is known as OPF - Optimal Power Flow [2]. The solution of an OPF also reveals the marginal prices in each one of the system buses (nodal prices), which can be used by the system operator and market agents as economic signals for congestion management [3].

Historically, the OPF was defined at the beginning of the 60's. The first method of solution proposed for OPF was the reduced gradient method or projected gradient method proposed by Carpentier in [4]. Later on, Dommel and Tinney [5] worked out the problem solving the Kuhn-Tucker equations using a combination of the gradient method for a known group of independent variables and penalty functions.

The general solution of the OFP consists on minimizing (or maximizing) an objective function subject to equality and inequality constraints. The objective function can take different forms depending on the purpose of OPF (to minimize losses, generation costs etc).

The OPF problem can be solved using a simplified model of the network, which leads to a DC Optimal Power Flow (DCOPF). The DC model of the network is obtained by ignoring losses and linearizing the power flow equations around a flat start where voltages in all buses are supposed to be the unit and angles have an initial value of zero. This simplified model had its origin in Stott and Alsac's studies in the OPF problem with security constraints. When constraints are not only considered under normal operation but also under contingencies, the OPF becomes a Security-Constrained Optimal Power Flow (SCOPF).

The consideration of feasibility after single contingencies appears in the literature in 1974 in the studies developed by Stott and Alsac [6]. Such consideration was implemented by means of an iterative process. The main disadvantage of the methodology proposed in [6] is that it does not guarantee the achievement of an optimal mathematical solution. To obtain an optimal solution it is necessary to consider all contingencies simultaneously; this is possible using coupled post-contingency power flows in the problem formulation as proposed in [7].

This paper presents the implementation of a contingency-based Security Constrained Optimal Power Flow model that reveals the marginal price of a blackout risk-equalizing policy used in the Colombian electricity market. The proposed methodology attempts to be a first approximation to the solution of the Colombian dispatch process which must ideally be carried out in a single stage.

II. THE DISPATCH PROCESS IN THE COLOMBIAN ELECTRICITY MARKET

The system operator of the Colombian power system (National Dispatch Center -CND) solves the energy dispatch

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process observing the rules of the Energy and Gas Regulatory Commission (CREG). CREG rules that the solution of the economic dispatch problem must be done in four different stages: ideal pre-dispatch, programmed pre-dispatch, preliminary programmed dispatch and programmed dispatch. Physical and reliability constraints of the electrical power system are added in each stage. The main disadvantage of solving the dispatch problem using a step by step methodology is that the nodal prices obtained are not accurate. The most particular characteristic in the process of the economic dispatch problem in Colombia is the inclusion of a blackout-risk equalizing policy. CND must find a solution of the dispatch problem so that the expected relative load shedding in the different sub-areas of the National Interconnected System (NIS) are equalized among the different sub-areas and minimized.

In Colombia a day ahead trading is carried out, every day generators make their price offers in pesos per megawatt-hour ($/MWh) and declare the available capacity of the generating units. With this information the CND solves the energy dispatch problem. This process is carried out in four different stages as ruled by CREG. These stages are shortly described below:

A. *Ideal Pre-dispatch*

In this stage the CND solves the energy dispatch process ignoring all network constraints; the expected demand is fulfilled dispatching generators according to price offers. This stage is used as a reference to identify the cost of electric constraints.

B. *Programmed Pre-dispatch*

In this stage the CND solves the energy dispatch process considering all network constraints along with maintenance schedules and minimum number of units on line per area to account for reliability criteria.

C. *Preliminary Programmed Dispatch*

In this stage the CND modifies the Programmed Pre-dispatch to include a reliability criterion in which the relative expected load shedding of the different sub-areas is equalized and minimized, in this case it is allowed to commit some units that had not yet been included in the Programmed Pre-dispatch.

D. *Programmed Dispatch*

Given the Preliminary Programmed Dispatch the CND calculates the programmed dispatch including Automatic Generation Control (AGC) requirements and verifies voltage stability conditions.

III. MATHEMATICAL FORMULATION

The following mathematical formulation corresponds to a contingency-based Security Constrained Optimal Power Flow model. This formulation is based on the use of coupled post-contingency Optimal Power Flows as proposed in [7] and includes the reliability criteria ruled by CREG by means of additional linear constraints.

In the formulation a unique system composed by islands is built. These islands correspond to the initial system (base case) and to the system after each of the contingencies. Dispatchable and curtailable loads, as well as shedded loads, are modeled as variable injections of negative power.

A. *Objective Function*

Equation (1) shows the objective function which is the cost of generation plus an additional variable that represents the difference between the expected relative loads shedding of a sub-area and the arithmetic average of all the expected relative loads shedding in the electrical power system. A high cost is assigned to this variable so that the load shedding among the different sub-areas are equalized.

$$\begin{gathered} Min\left[f(y) = f(P_g) + \; w_h h \right] \\ y = [\theta \;\; P_g \;\; h]^T \end{gathered} \tag{1}$$

Where

ng : Number of generators

nb : Number of buses

nj : Number of contingencies

$\theta : \left(\theta_i^j\right),\; i = 1\ldots nb \quad j = 0\ldots nj$

$P_g : \left(P_{gk}^{\;j}\right),\; k = 1\ldots ng \quad j = 0\ldots nj$

$f(P_g)$: Cost function of P_g

In this case j=0 corresponds to the base case, θ_i^j corresponds to the angle of bus i after contingency j and $P_{g\,k}^{j}$ corresponds to the output of generator k after contingency j. The probabilities of the contingencies have been used to weigh the cost of each possible operating outcome in the objective function as proposed in [8] making the cost a stochastic objective.

B. *Equality Constraints*

Equality constraints correspond to the DC power flow equations as shown in (2)

$$B^j \theta^j = M_g^{\;j} P_g^{\;j} - D^j, \quad j = 0...nj \tag{2}$$

Where

B^j : DC network matrix for contingency j

$M_g^{\;j}$: Generator positioning matrix for contingency j

D^j : Expected demand for contingency j

C. *Inequality constraints*

Inequality constraints correspond to generation limits (minimum and maximum), ramp limits and power flow limits in lines as shown in (3), (4) and (5) respectively.

$$P_g^{\;j-} \leq P_g^{\;j} \leq P_g^{\;j+} \tag{3}$$

$$L^{j-} \leq C^j \theta^j \leq L^{j+} \tag{4}$$

$$\left| P_{gk}^{\;\;j} - \; P_{gk}^{\;\;0} \right| \leq \Delta \tag{5}$$

Where

P_g^{j-} : Vector of minimum power generation limits for contingency j

P_g^{j+} : Vector of maximum power generation limits for contingency j

P_{gk}^{0} : Power output of generator k for the base case

P_{gk}^{j} : Power output of generator k for contingency j

L^{j-}, L^{j+} : Vector of line power flow limits in both directions for contingency j

C^{j} : Line susceptance matrix for contingency j

Δ : Vector of ramp limits

D. Additional linear constraints

To minimize and equalize the relative expected loads shedding among the different sub-areas as ruled by CREG, it is necessary to add some additional constraints to the problem formulation.

The expected load shedding in sub-area k due to contingency j is expressed in (6)

$$E[R_k^j] = P_s(j) R_k^j \tag{6}$$

Where

$E[R_k^j]$: Expected load shedding in sub-area k due to contingency j

$P_s(j)$: Probability that only contingency j takes place

R_k^j : Load shedding in sub-area k due to contingency j

The total relative expected loads shedding in sub-area k will be the sum of the expected loads shedding due to each of the contingencies divided by the total demand in the sub-area as expressed in (7).

$$\tilde{E}[R_k] = \frac{1}{dk} \sum_{j=1}^{nj} E[R_k^j] \tag{7}$$

Where

$\tilde{E}[R_k]$: Total relative expected loads shedding in sub-area k

dk : Total demand in sub-area k

The arithmetic average of the relative expected loads shedding in the different sub-areas is given by (8).

$$\rho = \frac{1}{nk} \sum_{k=1}^{nk} \tilde{E}[R_k] \tag{8}$$

Where

nk : Number of sub-areas

ρ : Arithmetic average of the expected loads shedding

$\tilde{E}[R_k]$: Total relative expected load shedding in sub-area k

An upper-bound constraint on the relative expected loads shedding in sub-area k can be expressed by (9)

$$\tilde{E}[R_k] \leq \rho + h \tag{9}$$

which can be written as:

$$-\sum_{\substack{j=1 \\ j\neq k}}^{nk} \frac{1}{nk} \tilde{E}[R_j] + \left(1 - \frac{1}{nk}\right) \tilde{E}[R_k] \leq h \tag{10}$$

Equation (10) can be expressed as a function of the optimization variables and represents the difference between the relative expected loads shedding of a sub-area and the arithmetic average. Variable *h* is given a high cost in the objective function, so that it is minimized and the relative expected loads shedding among the different sub-areas are equalized. Variable *h* can also be handled as a fixed parameter; in this case an additional equality constraint is added to the formulation, and the Lagrange multiplier associated with this constraint at the solution reveals the marginal cost of the risk-equalization policy.

IV. RESULTS USING A PROTOTYPE OF THE COLOMBIAN POWER SYSTEM

The proposed methodology has been applied on a prototype of the Colombian power system with 20 buses and 13 generators shown in Fig 1. The data for the prototype system have been provided by CND and are included in the Appendix. To solve the optimization problem the software Matpower [9] has been used.

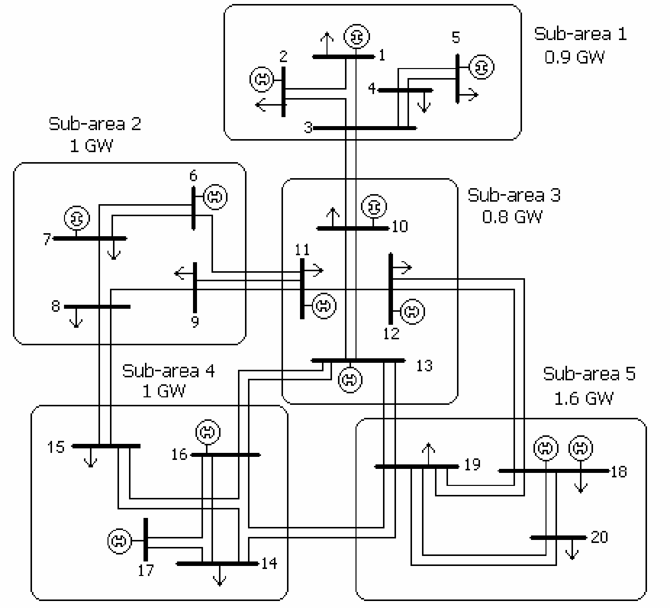

Fig. 1. Prototype of the Colombian power system with 20 buses and 13 generators.

The loads modeled as variable injections of negative power have been assigned a lost-load price of 3220.318 $/kWh as dictated by the Mining and Energy Planning Unit (UPME) for April 2006. The marginal cost curves handling h as a variable and as a parameter are shown in Fig 2 and Fig 3 respectively. The slight differences arise from tolerances in the linear programming solver. The structure of the equality and inequality constraints matrices are shown in Fig 4 and Fig 5 respectively.

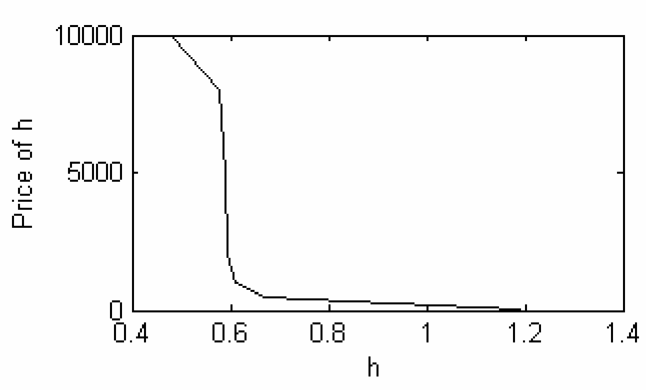

Fig. 2. Marginal cost curve using h as a variable

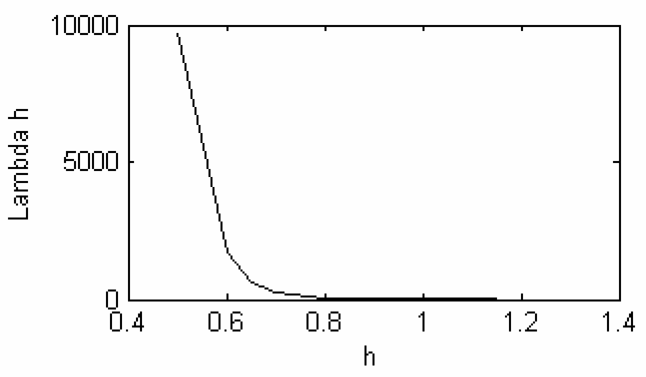

Fig. 3. Marginal cost curve using h as a parameter

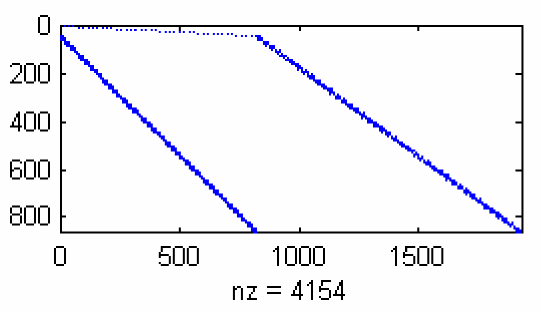

Fig. 4. Structure of the matrix of equality constraints

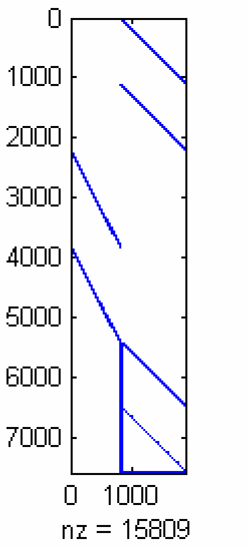

Fig. 5. Structure of the matrix of inequality constraints

V. CONCLUSIONS

A contingency-based Security Constrained Optimal Power Flow model has been applied on a prototype of the Colombian power system to reveal the marginal price of a black-out risk equalizing policy in the Colombian electricity market. The main contribution of this paper is the inclusion of the reliability criteria ruled by CREG in the optimization process, and the solution of the Colombian dispatch problem in one single stage which provides actual nodal prices. Further work would improve on the methodology by employing an AC network model.

VI. APPENDIX

Table I shows the generator data for the power system shown in Fig 1. Table II shows the branch data for the power system shown in Fig 1, where L.C and P.C stand for line capacity and probability of contingency respectively. Table III shows the load data for the power system shown in Fig 1.

TABLE I
GENERATOR DATA FOR THE POWER SYSTEM SHOWN IN FIG 1

Gen	Bus	Min (MW)	Max (MW)	Bid ($/MWh)
G1	1	20	750	180000
G2	2	10	100	170000
G3	5	10	300	160000
G4	6	100	500	60000
G5	7	50	400	50000
G6	10	50	200	80000
G7	11	100	800	90000
G8	12	20	250	185000
G9	13	100	1200	45000
G10	16	50	350	175000
G11	17	80	600	85000
G12	18	100	1000	42000
G13	18	110	1100	40000

TABLE II
BRANCH DATA FOR THE POWER SYSTEM SHOWN IN FIG 1

From bus	To bus	r (pu)	x (pu)	b (pu)	L.C (MW)	P.C (%)
1	3	0.00618	0.0421	0.06184	216	8.332
2	12	0.00834	0.0483	0.06184	288	3.743
3	3	0.00493	0.0493	0.02672	333	1.892
3	4	0.00119	0.1932	0.01187	225	13.250
3	4	0.00118	0.1845	0.00841	333	0.147
3	10	0.00722	0.0769	0.07228	1900	0.496
4	10	0.00525	0.0762	0.07293	1900	23.123
4	5	0.00589	0.1892	0.07293	225	10.475
6	5	0.01282	0.1892	0.12851	225	26.089
6	7	0.00548	0.0628	0.00547	282	23.213
6	7	0.00548	0.0562	0.00544	266	9.277
7	11	0.00548	0.0408	0.00544	315	1.839
8	8	0.00997	0.0683	0.13328	314	21.059
8	9	0.01484	0.0387	0.14171	266	1.896
8	15	0.12765	0.1258	0.15467	343	20.548
9	15	0.01038	0.1258	0.15639	343	1.893
9	11	0.01378	0.0379	0.13819	266	1.568
10	11	0.01378	0.0379	0.13819	266	5.824
10	13	0.14750	0.0465	0.14783	1900	10.423
11	13	0.14750	0.0465	0.14783	1900	12.956
12	13	0.01523	0.0891	0.15265	330	0.952
12	18	0.01523	0.0324	0.15265	330	0.952
13	18	0.01827	0.0324	0.17606	330	3.705
13	16	0.01755	0.0985	0.07274	307	0.952
13	16	0.01228	0.0945	0.25428	274	0.952
13	19	0.02940	0.0962	0.13921	332	0.952
13	19	0.01940	0.0962	0.13921	332	27.308
15	16	0.02122	0.0206	0.01774	275	13.236
15	14	0.02106	0.0212	0.01435	250	10.123
16	19	0.00510	0.1254	0.07282	250	0.952
17	16	0.01224	0.0547	0.00326	373	0.952
17	14	0.00602	0.0493	0.08621	364	5.266
14	16	0.00585	0.0477	0.08054	364	0.952
14	19	0.00591	0.1254	0.08088	250	0.952
19	18	0.00317	0.0312	0.04385	343	0.952
19	18	0.00317	0.0312	0.04385	343	0.952
19	20	0.00758	0.0326	0.10482	329	14.335
19	20	0.00758	0.0326	0.10482	350	0.952
20	18	0.01012	0.0225	0.01088	329	0.952
20	18	0.01012	0.0225	0.01088	329	2.178

TABLE III
LOAD DATA FOR THE POWER SYSTEM SHOWN IN FIG 1

Bus	Sub-area	Load (MW)
1	1	200
2	1	200
4	1	300
5	1	100
7	2	300
8	2	350
9	2	350
10	3	500
12	3	300
14	3	500
15	4	350
18	4	300
19	5	800
20	5	500

VII. ACKNOWLEDGMENT

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